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TEACHING MATERIAL ON

MATHEMATICS

SCHOOL OF SCIENCE

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Module-1 (39) [1]

Classification of Optimization Problems:

The idea about classification of optimization problem is based on the following:—

(i) the type of constraints
(ii) the nature of designed variable
(iii) the physical structure of the problem
(iv) the nature of the equations involved
(v) deterministic nature of the design variable
(vi) separability of the function and nature of objective functions.

The above classifications are explained briefly as under:-

(1) Classification based on existence or type of constraints.

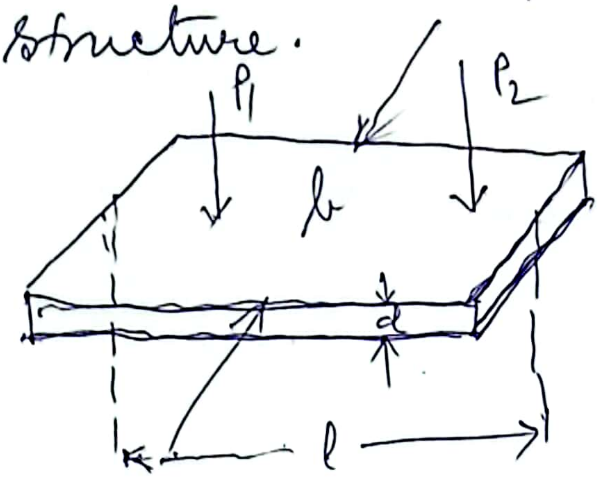
(a) Constrained optimization problems are subject to one or more constraints
(b) Unconstrained optimization problems There does not exists any constraints equation

(2) Classification based on the nature of designed variables:-

There are two broad categories of classification within the classification itself i.e.

Category 1st The objective is to find set of design parameters that make a prescribed function of these parameters minimum or maximum subject to certain constraints.

For example to find the minimum weight design of a strip footing with two loads shown in the figure below subject to a limitation on the maximum settlement of the structure.



The formulation of problem can be as under:-

Find $X = \begin{Bmatrix} b \\ d \end{Bmatrix}$ which minimizes

$$f(X) = \bar{h}\,(b, d)$$

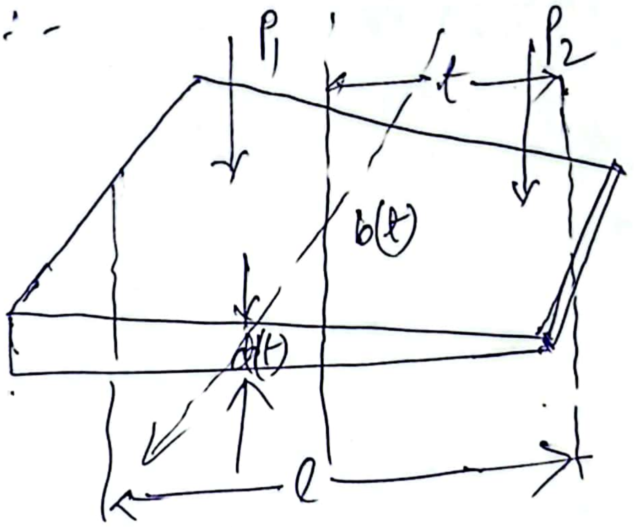
subject to the constraints $\delta_s(X) \le \delta_{max}$

$$b \ge 0$$
$$d \ge 0$$

The length of the footing (l) the loads P_1 and P_2, the distance between the loads are assumed to be constant. and the required optimization is achieved by varying b and d. Such problems are called parameter or static optimization problems.

2nd Category:- The objective is to find a set of designed parameters which are all continuous functions of some other parameters, that minimizes an objective function subject to a set of constraints.

For example:- If the cross-sectional dimension of the rectangular footing is allowed to vary along its length as shown in the figure below:-



The problem can be defined as follows:-

Find $\bar{X}(t) = \begin{Bmatrix} b(t) \\ d(t) \end{Bmatrix}$ which minimizes

$$f(\bar{X}) = \bar{g}(b(t), d(t))$$

$- C_3 - C_0 Y_3 = 4 - (0,0,0)(0,5,4) - 4$

subject to the (42) constraints

$$\delta_i(\bar{x}(t)) \le \delta_{max} \; ; \; 0 \le t \le l$$

$$h(t) \ge 0 \; ; \; 0 \le t \le l$$

$$d(t) \ge 0 \; ; \; 0 \le t \le l$$

The length of the following (l) the loads P_1 and P_2 the distance between the loads are assumed to be constant and the required optimization is called as dynamic optimization problems and is achieved by varying h and d. It is also known as "trajectories"

(3) Classification based on the physical structure of the problems:—

Based on the physical structure of the problem we can classify optimization problems in two two kinds one is optimal control and non-optimal control problems.

(1) An optimal Control (OC) problem is a mathematical programming problem involving a number of stages where each stage evolves from the preceding stage in a prescribed manner.

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→ It is defined by (43) two types of variables :— the control or design variables and state variables.

→ The problem is to find a set of control or design variables such that the total objective function (also known as the performance index, PI) over all stages is minimized subject to a set of constraints on the control and state variables. An OC problem can be stated as follows:

Find $\bar{X}$ which minimizes

$$f(\bar{X}) = \sum_{i=1}^{l} f_i(x_i, y_i)$$

Subject to the constraints

$$q_i(x_i, y_i) + y_i = y_{i+1} \; ; \; i = 1, 2 \ldots l$$

$$g_j(x_j) \leq 0 \; ; \; j = 1, 2, \ldots l,$$

$$h_k(y_k) \leq 0 \; ; \; k = 1, 2, \ldots l,$$

where x_i is the ith control variable, y_i is the ith state variable, and f_i is the contribution of the ith stage to the total objective function. g_j, h_k, and q_i are the function of x_j, y_j; x_k, y_k and x_i and y_i respectively.

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and l is the total number of states and state variables x_i and y_j can be vectors in some cases.

(ii) The problems which are not optimal control problems are called non-optimal control problems.

4) Classification based on the nature of the equations involved:—

→ Based on the nature of expressions for the objective function and the constraints optimizations problems can be classified as Linear, Non-linear, Geometric and Quadratic programming problems.

(a) Linear Programming Problems:—

If the objective function and all the constraints are linear functions of the design variables, the mathematical programming problems LP is called a linear programming problem (LP) Problem often stated in the standard form

$$\text{Find } X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} \quad \text{S.T} \sum_{i=1}^{n} a_{ij} x_i = b_j;\ x_i \geq 0$$

which minimizes $f(X) = \sum_{i=1}^{n} c_i x_i$, where c_i, a_{ij} and b_j are constants.

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(b) Non-linear programming problems :– (B-6) 23

* If any of the function among the objectives and constraints function is non-linear, the problem is called a non-linear programming (NLP) problem. This is the most general form of a programming problem.

(c) Geometric programming (GMP) problems :– is one in which objective function and constraints are expressed as ~~polynomials in X~~ posynomial in $\vec{X}$.

A posynomial with N terms can be expressed as
$$h(X) = c_1 x_1^{a_{11}} x_2^{a_{12}} \cdots x_n^{a_{1n}} + \cdots + c_N x_1^{a_{N1}} x_2^{a_{N2}} \cdots x_n^{a_{Nn}},$$

Thus GMP problems can be expressed as follows :–

Find $\vec{X}$ which minimizes
$$f(X) = \sum_{i=1}^{N_0} c_i \left(\prod_{j=1}^{n} x_j^{p_{ij}} \right), \quad c_i > 0, x_j > 0$$

$$g_k(X) = \sum_{i=1}^{N_k} a_{ik} \left(\prod_{j=1}^{n} x_j^{q_{ij}} \right) > 0, \quad a_{ik} > 0, x_j > 0$$
and $k = 1, 2, \ldots, m$

where N_0 and N_k denote the number of terms in the objective and k^{th} constraint function respectively.

• Where, x_j is the jth control variable, y_i is the ith state variable, and f_i is the contribution of the ith stage to the total objective function. g_j, h_k, and q_i are the functions of x_j, y_j; x_k, y_k and x_i and y_i, respectively, and l is the total number of states. The control and state variables x_j and y_j can be vectors in some cases.

(ii) The problems which are not optimal control problems are called non-optimal control problems.

(4) <u>Classification based on the nature of the equations involved:—</u>

• Based on the nature of expressions for the objective function and the constraints, optimization problems can be classified as linear, non-linear, geometric and quadratic programming problems.

~~(5) Deterministic nature of the variables~~

(a) <u>Linear programming problem:—</u>

* If the objective function and all the constraints are linear functions of the design variables, the mathematical programming problem is called a linear programming (LP) problem often stated in the standard form

$$\text{Find } X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}, \quad \text{s.t. } \sum_{i=1}^{n} a_{ij} x_i = b_j, \quad x_i \geq 0.$$

~~where~~

which maximizes $f(X) = \sum_{i=1}^{n} c_i x_i$

where c_i, a_{ij} and b_j are constants

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(d) Quadratic programming problems:

* A quadratic programming problem is the ~~best~~ non-linear programming problem with a quadratic objective function and linear constraints and is concave (for maximization problems). It is usually formulated as follows:—

Find $\vec{x}$ which minimizes,

$$f(\vec{x}) = c + \sum_{i=1}^{n} q_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} Q_{ij} x_i x_j$$

Subject to $\sum_{i=1}^{n} a_{ij} x_i = b_j$, $j = 1, 2, \ldots, m$
$i = 1, 2, \ldots n$

and $x_i \geq 0$

Where c, q_i, Q_{ij}, a_{ij} and b_j are constants

(V) Classification based on deterministic nature of the variables:—

* Under this classification optimization problems can be classified as (a) deterministic and (b) stochastic programming problems.

(a) Deterministic programming problems:—
In this type of problems all the designed variables are deterministic (which can be determined exactly).

(b) Stochastic programming problems:—
In this type of an optimization problem some or all the parameters (design variables and/or pre-assigned parameters) are probabilistic (non-deterministic or stochastic).

For example estimates of life span of (Pg-8) structures which have probabilistic inputs of the concrete strength and load capacity. A deterministic value of the life-span is non-attainable.

(VI) Classification based on the permissible values of the decision variables :—

* Under this classification, problems can be seggregated into (a) integer and (b) real valued programming problems

(a) Integer programming problems :—

* If some or all of the design variables are restricted to take only integer (or discrete) values only, then such an optimization problem is called an integer programming problem.

(b) Real-valued programming problem :—

* A real valued problem is that in which it is sought to minimize or maximize a real function by systematically choosing the values of real variables from within an allowed set. When the allowed set contains only real values it is called a real-valued programming problem.

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(VII) Classification based on separability of the functions:— (Pg 24(b))

- Based on the separability of the objective and constraint functions, optimization problems can be categorize into (a) Separable and (b) non-separable programming problems

(a) Separable programming problems:—

- In this type of a problem the objective function and the constraints are separable. A function is said to be separable if it can be expressed as the sum of n single-variable functions and such a separable programming problems can be expressed in standard form as

Find X which minimizes $f(X) = \sum_{i=1}^{n} f_i(x_i)$

subject to: $g_j(X) = \sum_{i=1}^{n} g_{ij}(x_i) \le b_j$, $j = 1, 2, \ldots m$

where b_j is a constant.

(III) (4) Classification based (57) on the number of objective functions :— Under this classification objective functions can be classified as (i) single and (ii) multiobjective programming problems.

(i) Single - objective Prog. problems ~~are which~~ those in which there is only single objective.

(ii) Multi-objective programming problems ~~are~~ can be stated as follows :→

Find X which minimizes $f_1(X), f_2(X), f_3(X) \ldots f_K(X)$

subject to : $g_j(X) \leq 0$, $j = 1, 2, \ldots, m$.

where $f_1, f_2, \ldots f_K$ are the objective functions to be minimized simultaneously.

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